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Fully-adaptive Model for Broadcasting with Universal Lists

Saber Gholami

SYNASC 2022
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Linz, Austria.

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Introduction

- Computer networks are becoming more popular each day!
- One problem: Propagate a message
- Information dissemination:
  - Unicasting,
  - **Broadcasting**,
  - Multicasting,
  - ...

Introduction - cont.

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- Broadcasting is the process of distributing a message from a single node (*originator*) to all other nodes of the network,
- Each *call* is performed during one unit of time,
- Several *calls* could be performed in parallel.

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- The network: $G = (V, E)$, originator $u \in V$.

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- The network: $G = (V, E)$, originator $u \in V$.
- $B_{cl}(u, G)$: minimum time required to finish the broadcasting from u.
- $B_{cl}(G) = \max\{B_{cl}(u, G) | u \in V(G)\}$
  - For any graph: $B_{cl}(G) \geq \lceil \log n \rceil$

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- $B_{cl}(u, G)$: minimum time required to finish the broadcasting from u.
- $B_{cl}(G) = \max\{B_{cl}(u, G) | u \in V(G)\}$
  - For any graph: $B_{cl}(G) \geq \lceil \log n \rceil$
- NP-Complete in arbitrary graphs,
- NP-Hard to approximate within a ratio of $(3 - \varepsilon)$ for any $\varepsilon > 0$.
- Solved optimally for only a few networks,
- A long list of heuristics and approximation algorithms!

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- Broadcast scheme: the ordering of the neighbors of each vertex, depending on the originator:
  - Fix vertex u as the originator,
  - once vertex v gets informed, it will follow its list l_v^u,

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  - Fix vertex u as the originator,
  - once vertex v gets informed, it will follow its list l_v^u,
- Drawbacks:
  - Each vertex has to maintain up to $|V|$ different lists for different originators,
  - A vertex has to know the originator to perform broadcasting.
  - Requires a comprehensive knowledge of the network for every vertex.

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- Drawbacks:
  - Each vertex has to maintain up to $|V|$ different lists for different originators,
  - A vertex has to know the originator to perform broadcasting.
  - Requires a comprehensive knowledge of the network for every vertex.
  - Inefficient in real-world networks:
    - Increased message bits,
    - Need for larger local memory.

Preliminaries - Universal lists broadcasting

- Broadcasting with universal lists:
  - Each vertex v has a single list l_v to follow, regardless of the originator.

[1] Slater, P.J., Cockayne, E.J. and Hedetniemi, S.T., 1981. Information dissemination in trees. *SIAM Journal on Computing*, 10(4), pp.692-701..
[2] Diks, K. and Pelc, A., 1996. Broadcasting with universal lists. *Networks*, 27(3), pp.183-196.

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- Broadcasting with universal lists:
  - Each vertex v has a single list l_v to follow, regardless of the originator.
- Two sub-models:
  - **Non-adaptive** $B_{na}(G)$: send to all vertices on the list,
  - **Adaptive** $B_a(G)$: skip the vertices from which the message is received.

[1] Slater, P.J., Cockayne, E.J. and Hedetniemi, S.T., 1981. Information dissemination in trees. *SIAM Journal on Computing*, 10(4), pp.692-701..
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Preliminaries - Universal lists broadcasting

- Broadcasting with universal lists:
  - Each vertex v has a single list l_v to follow, regardless of the originator.
- Two sub-models:
  - **Non-adaptive $B_{na}(G)$:** send to all vertices on the list,
  - **Adaptive $B_a(G)$:** skip the vertices from which the message is received.
- Introduced indirectly by Slater [1]; for any Tree, $B_{cl}(T) = B_a(T)$.
- Diks and Pelc [2] distinguished between adaptive and non-adaptive models,
  - Also proposed several broadcast schemes for different graphs
- Long list of research ...

[1] Slater, P.J., Cockayne, E.J. and Hedetniemi, S.T., 1981. Information dissemination in trees. *SIAM Journal on Computing*, 10(4), pp.692-701..
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- Another sub-model for universal lists,
- A universal list l_u is maintained at each vertex u,
- Once informed, follow the list and **skip all informed vertices!**
  - Similarly to the classical model: No unnecessary calls!

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- Another sub-model for universal lists,
- A universal list l_u is maintained at each vertex u,
- Once informed, follow the list and **skip all informed vertices!**
  - ◇ Similarly to the classical model: No unnecessary calls!
- **Theorem 1.** $B_{cl}(G) \leq B_{fa}(G) \leq B_a(G) \leq B_{na}(G)$, *for any graph* G.
-

Model	Symbol	No. of unnecessary calls	Space Complexity	Speed
Non-adaptive	$B_{na}(G)$	Many	Very Low	Very Slow
Adaptive	$B_a(G)$	Few	Low	Slow
Fully Adaptive	$B_{fa}(G)$	0	Moderate	Moderate
Classical	$B_{cl}(G)$	0	Very High	Very Fast

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- A broadcast scheme: Matrix $\sigma_{n \times \Delta}$,
  - Row i of σ corresponds to an ordering for vertex v_i.
- Set of all possible schemes: Σ.

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- A broadcast scheme: Matrix $\sigma_{n \times \Delta}$,
  - Row i of σ corresponds to an ordering for vertex v_i.
- Set of all possible schemes: Σ.
- Let $M \in \{na, a, fa\}$ be a model:
  - $B_M^\sigma(v, G)$: the time steps needed to inform all the vertices in G from v while following σ under M,
  - $B_M^\sigma(G) = \max_{v \in V}\{B_M^\sigma(v, G)\}$,
  - $B_M(G) = \min_{\sigma \in \Sigma}\{B_M^\sigma(G)\}$.

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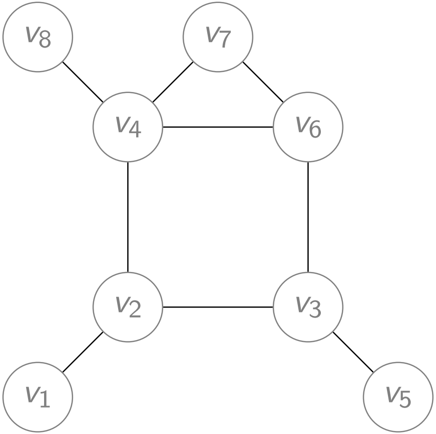
Sender	Ordering of receivers			
v_1	v_2	Null	Null	Null
v_2	v_3	v_4	v_1	Null
v_3	v_2	v_6	v_5	Null
v_4	v_2	v_6	v_8	v_7
v_5	v_3	Null	Null	Null
v_6	v_3	v_7	v_4	Null
v_7	v_6	v_4	Null	Null
v_8	v_4	Null	Null	Null



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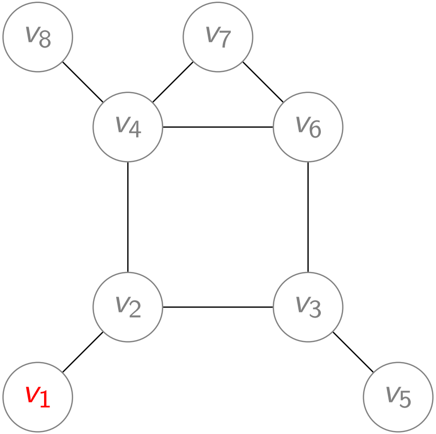
v_6	v_3	v_7	v_4	Null
v_7	v_6	v_4	Null	Null
v_8	v_4	Null	Null	Null



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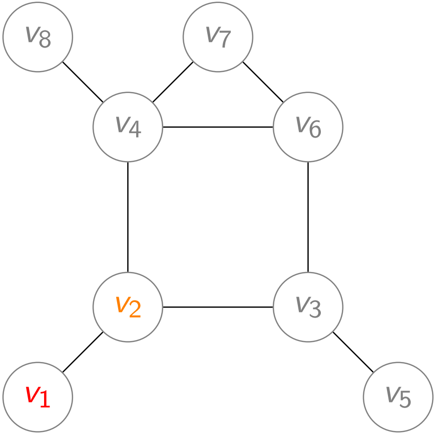

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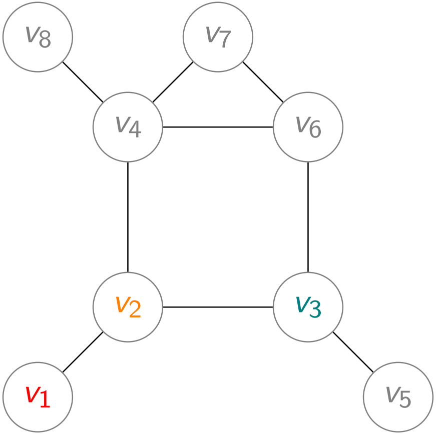

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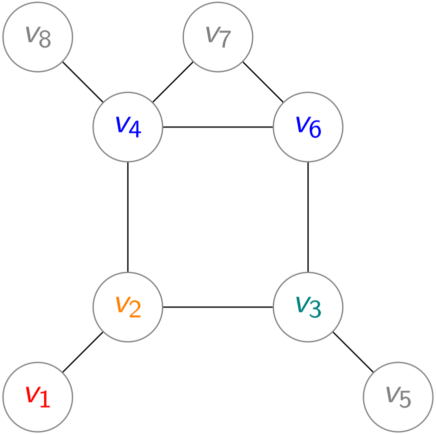

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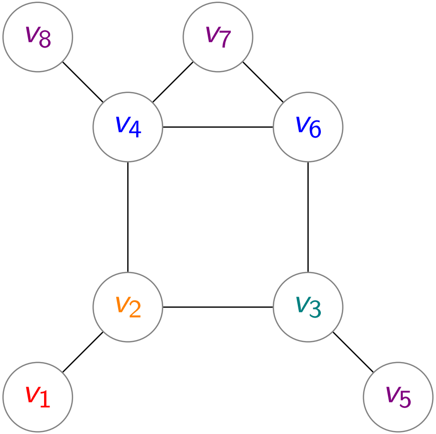

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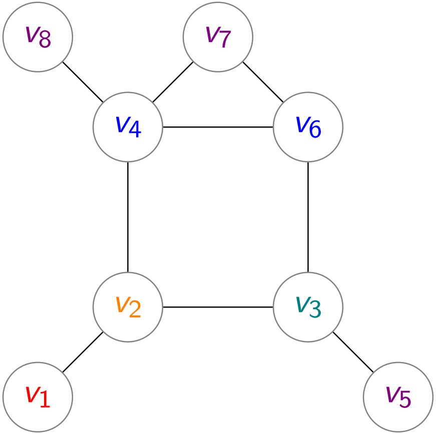
v_8	v_4	Null	Null	Null



- $B^{\sigma}_{fa}(v_1, G) = 4$, while $B^{\sigma}_{a}(v_1, G) = 5$ and $B^{\sigma}_{na}(v_1, G) = 6$.

Fully-adaptive Model - AAA

- **Assumptions:**
  - None-faulty network with established links,
  - Unique and heavy message,
  - The message: header + payload,

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- **Assumptions:**
  - None-faulty network with established links,
  - Unique and heavy message,
  - The message: header + payload,
- **Architecture:**
  - How to know the state of each neighbour?
    - Push model,
    - Pull model,

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- **Assumptions:**
  - None-faulty network with established links,
  - Unique and heavy message,
  - The message: header + payload,
- **Architecture:**
  - How to know the state of each neighbour?
    - Push model,
    - Pull model,
- **Applications:**
  - Update procedure in SDNs:
    - Changing routing policies, adjusting links' weights, etc.
    - The data plane only forwards packets,
    - Routing and load balancing decisions are made in a centralized controller,
    - The network manager must optimize the forwarding tables (broadcast schemes).

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- Trees T:
  - **Theorem 2.** $B_{cl}(T) = B_{fa}(T) = B_a(T)$.

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- Grids $G_{m \times n}$:
  - **Corollary 1.**
    $B_{fa}(G_{m \times n}) = m + n - 2.$

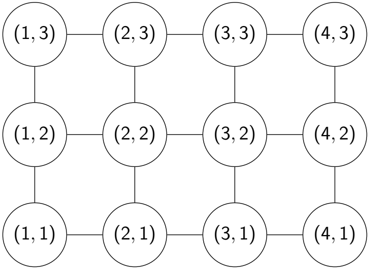

Figure: Grid with $m = 4, n = 3$

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- Tori $T_{m \times n}$:
  - **Theorem 3.**
  - $B_{fa}(T_{m\times n}) = \lfloor \frac{n}{2} \rfloor + \lfloor \frac{m}{2} \rfloor$, if m and n are even,
  - $B_{fa}(T_{m\times n}) = \lfloor \frac{n}{2} \rfloor + \lfloor \frac{m}{2} \rfloor + 1$, if only one of m and n is even,
  - $\lfloor \frac{n}{2} \rfloor + \lfloor \frac{m}{2} \rfloor + 1 \le B_{fa}(T_{m\times n}) \le \lfloor \frac{n}{2} \rfloor + \lfloor \frac{m}{2} \rfloor + 2$, if m and n are odd.

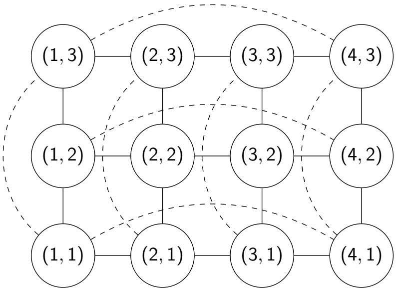

Figure: Torus with $m = 4, n = 3$

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- Cube Connected Cycle CCC_d:
  - **Theorem 4.**
  - $B_{fa}(CCC_d) = \lceil \frac{5d}{2} \rceil - 1$

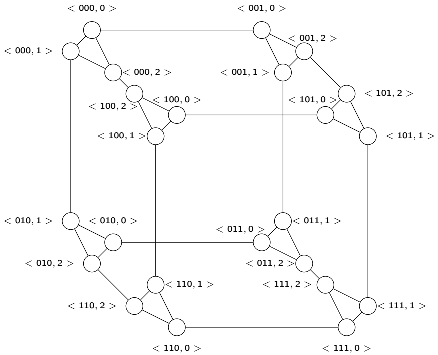


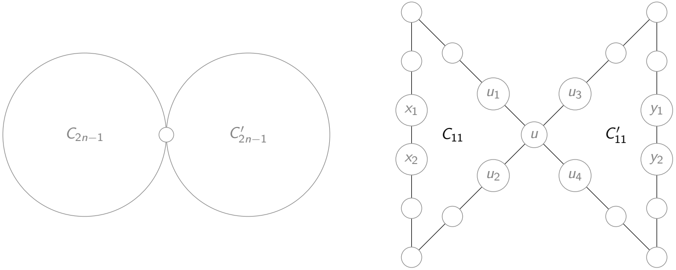
Figure: Cube-Connected Cycle with $d = 3$

Results on the Fully-adaptive Model - cont.

- Is $B_{cl}(G) = B_{fa}(G)$ always?

Results on the Fully-adaptive Model - cont.

- Is $B_{cl}(G) = B_{fa}(G)$ always?
  - No!
  - **Proposition 1.** *There exists graph G with $B_{cl}(G) < B_{fa}(G)$:*



a) b)

Outline

1. Introduction
2. Preliminaries and Literature Review
3. Fully-adaptive Model
4. Results on the Fully-adaptive Model
   - Trees
   - Grids $G_{m \times n}$
   - Tori $T_{m \times n}$
   - Cube Connected Cycle CCC_d
   - Graphs with $B_{cl}(G) < B_{fa}(G)$
5. Conclusion and Future works

Conclusion

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Conclusion and Future works

- We proposed the Fully-adaptive model for broadcasting with universal lists
- Benefits:
  - Uses less memory compared to classical broadcasting,
  - Faster than the adaptive and non-adaptive models,
- We designed optimal broadcast schemes for:
  - Trees,
  - Grids,
  - Tori,
  - Cube-connected Cycle,
- We designed graphs with $B_{cl}(G) < B_{fa}(G)$.

Future works

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Conclusion and Future works

- Finding the broadcast time of other families of graphs under the fully-adaptive model, such as:
  - Complete graph,
  - Shuffle-exchange,
  - DeBruijn,
  - Complete bipartite graph,
  - etc.
- Designing an efficient algorithm to find $B_{fa}(G)$ for general graphs, or prove that it is NP-Hard.


THANK YOU
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